DEFORMATION AND FAILURE UNDER IMPACT LOADING. MODEL OF A THERMOELASTOPLASTIC MEDIUM

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Various models are used in the numerical analysis of structural elements for an intense pulsed effect. There is extensive use in the case of adiabatic approximation of an elastoplastic flow model of the Prandtl–Reis type and the Wilkins method is used for its numerical realization [1, 2]. Recently on the basis of thermodynamic principles of solid mechanics there has been rapid development of models of solids with internal parameters of state by means of which deformation processes and so-called continuous failure are described ([3-8], etc.). In models with internal parameters of state it is normal to suggest that failure occurs in the case when the value of some parameter reaches a critical magnitude [6, 9-13]. A simple version of this model (acoustic approximation) is obtained if it is assumed that the internal parameter only characterizes the process of continuous failure, i.e., damage accumulation, and it does not affect the deformation process in the material since the internal parameter does not enter into the fundamental equations for the medium [9]. More complex models take account of the reciprocal effect of deformation processes, damage accumulation, and temperature effects [6]. In the present work on the basis of thermodynamic principles of solid mechanics a coherent model is built up for a damaged thermoelastoplastic body with internal parameters which is a development of the flow model [1, 2]; criteria are suggested for failure of limiting specific dissipation; use of the model is considered for describing spalling failure with impact of metal plates.

1. Model of the Material. We introduce the following values: tensor components for stresses σ_{ij}, strains ε_{ij}, elastic ε^e_{ij} and plastic ε^p_{ij} strains $(\varepsilon_{ij}=\varepsilon^e_{ij}+\varepsilon^p_{ij})$; specific (per unit mass) free energy F, internal energy U, and entropy η, and also absolute temperature T, thermal flow $\mathbf{q}$, density ρ_0, and structural parameter ω. Here it is assumed that ω describes initiation and growth of damage for the material during deformation.

We turn to an equation for internal energy written in the form of heat inflow:

$$\dot{U}=(1/\rho_0)\sigma_{ij}\dot{\varepsilon}_{ij}-(1/\rho_0)\operatorname{div}\mathbf{q}. \tag{1.1}$$

Here and subsequently a full stop above a symbol signifies a derivative with respect to time along the trajectory of a solid particle.

We present the second law of thermodynamics in the form of a Clausius–Duhem inequality

$$\dot{\eta}\geqslant-(1/\rho_0)\operatorname{div}(\mathbf{q}/T). \tag{1.2}$$

By comparing (1.1) and (1.2) and changing over from internal energy to free energy

$$F=U-\eta T, \tag{1.3}$$

and from the Clausius–Duhem inequality we obtain

$$\frac{1}{\rho_0}\sigma_{ij}\dot{\varepsilon}_{ij}-\frac{1}{\rho_0}\frac{\mathbf{q}\operatorname{grad}T}{T}-\dot{T}\eta-\dot{F}\geqslant 0. \tag{1.4}$$

considering that F is a function of the independent variables ε^e_{ij}, ε^p_{ij}, ω, and T, we arrive from (1.4) to

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$$\left(\frac{1}{\rho_0}\sigma_{ij}-\frac{\partial F}{\partial \varepsilon^e_{ij}}\right)\dot{\varepsilon}^e_{ij}-\left(\frac{\partial F}{\partial T}+\eta\right)\dot{T}+\frac{1}{\rho_0}\sigma_{ij}\dot{\varepsilon}^p_{ij}-\frac{\partial F}{\partial \omega}\dot{\omega}-\frac{\partial F}{\partial \varepsilon^p_{ij}}\dot{\varepsilon}^p_{ij}-$$
$$-\frac{1}{\rho_0}\frac{\mathbf{q}\,\mathrm{grad}\,T}{T}\geqslant 0. \qquad (1.5)$$

As shown in [3], from (1.5) it follows that

$$\sigma_{ij}=\rho_0\frac{\partial F}{\partial \varepsilon^e_{ij}},\quad \eta=-\frac{\partial F}{\partial T}, \qquad (1.6)$$

and then (1.5) is transformed to the form

$$d\equiv\left(\sigma_{ij}-\rho_0\frac{\partial F}{\partial \varepsilon^p_{ij}}\right)\dot{\varepsilon}^p_{ij}-\rho_0\frac{\partial F}{\partial \omega}\dot{\omega}-\frac{\mathbf{q}\,\mathrm{grad}\,T}{T}\geqslant 0, \qquad (1.7)$$

where $d \equiv \rho_0\gamma$ is a function of dissipation (γ is production of entropy).

We turn again to Eq. (1.1) which by means of (1.3) may be written as

$$\dot{F}+\dot{\eta}T+\eta\dot{T}=\frac{1}{\rho_0}\sigma_{ij}\dot{\varepsilon}^e_{ij}+\frac{1}{\rho_0}\sigma_{ij}\dot{\varepsilon}^p_{ij}-\frac{1}{\rho_0}\mathrm{div}\,\mathbf{q}.$$

By using (1.6) finally we obtain

$$\dot{\eta}T=\left(\frac{1}{\rho_0}\sigma_{ij}-\frac{\partial F}{\partial \varepsilon^p_{ij}}\right)\dot{\varepsilon}^p_{ij}-\frac{\partial F}{\partial \omega}\dot{\omega}-\frac{1}{\rho_0}\mathrm{div}\,\mathbf{q}. \qquad (1.8)$$

In thermal conductivity Eq. (1.8) we define entropy as a function of its independent variables $\varepsilon^e_{ij}, \varepsilon^p_{ij}, \omega, T$. For this we make the following simplifying assumptions. A. Strains are small: square terms in the expression for the strain tensor are ignored. B. Free energy is presented in the form of the sum of two terms:

$$F=F_1(\varepsilon^e_{ij},\omega,T)+F_2(\varepsilon^p_{ij},\omega,T). \qquad (1.9)$$

The first of the relationships in (1.6) taking account of (1.9) gives $\sigma_{ij}=\rho_0\partial F_1/\partial\varepsilon^e_{ij}$. Among the arguments of function F_1 there are no plastic strain tensor components, and therefore hypothesis (1.9) is equivalent to assuming that accumulation of plastic strain does not change the elastic properties of the material. C. Dissipation function d (1.7) is written as the sum of three non-negative terms, and in fact:

$$d_M=t_{ij}\dot{\varepsilon}^p_{ij}\geqslant 0,\quad d_f=-\rho_0\frac{\partial F}{\partial \omega}\dot{\omega}\geqslant 0,\quad d_T=-\frac{\mathbf{q}\,\mathrm{grad}\,T}{T}\geqslant 0 \qquad (1.10)$$

(d_M is mechanical dissipation, d_f is dissipation of continual failure, d_T is thermal dissipation). In addition, we denote

$$t_{ij}=\sigma_{ij}-\rho_0\partial F/\partial\varepsilon^p_{ij}. \qquad (1.11)$$

Tensor t_{ij} is called the active stress tensor. It follows from (1.10) and (1.11) that if free energy depends on plastic strain ε^p_{ij}, then the process of energy dissipation is determined not by true stresses σ_{ij}, but by "active" stresses t_{ij}. Introduction of ε^p_{ij} into free energy makes it possible to model the strain anisotropy of the material which arises with plastic deformation.

Concerning the dissipation of continuous failure it is assumed that

$$-\rho_0\partial F/\partial\omega=\Lambda\dot{\omega} \qquad (1.12)$$

($\Lambda \geq 0$ is a material parameter). It is noted that with Λ = const relationship (1.12) is the result of Onsager theory [14].

Apart from free energy we introduce thermodynamic potential

$$G = F - (1/\rho_0)\,\sigma_{ij}\varepsilon^e_{ij}. \tag{1.13}$$

Differentiation of (1.13) with respect to time taking account of (1.6) gives

$$\rho_0 \dot{G} = -\sigma_{ij}\dot{\varepsilon}^e_{ij} - \rho_0 \eta \dot{T} + \rho_0 \frac{\partial F}{\partial \varepsilon^p_{ij}}\,\dot{\varepsilon}^p_{ij} + \rho_0 \frac{\partial F}{\partial \omega}\dot{\omega}. \tag{1.14}$$

We select an independent arguments of potential G σ_{ij}, ε^p_{ij}, and T. From (1.14) we obtain

$$\varepsilon^e_{ij} = -\rho_0 \frac{\partial F}{\partial \sigma_{ij}},\ \eta = -\frac{\partial G}{\partial T},\ \frac{\partial G}{\partial \varepsilon^p_{ij}} = \frac{\partial F}{\partial \varepsilon^p_{ij}},\ \frac{\partial G}{\partial \omega} = \frac{\partial F}{\partial \omega}. \tag{1.15}$$

By using the hypotheses A to C adopted the expression for G may be presented in the form

$$-\rho_0 G = \frac{2\mu - 3K}{36K\mu}\sigma^2_{kk} + \frac{1}{4\mu}\sigma_{ij}\sigma_{ij} + \frac{1}{3}\alpha_V \sigma_{kk}(T - T_0) + \\ + \frac{\Gamma}{2}\varepsilon^p_{ij}\varepsilon^p_{ij} + \Lambda \int_0^{\omega} \dot{\omega}\, d\omega + G_0(T) \tag{1.16}$$

(α_V, μ, and K are coefficients of volumetric expansion, shear modulus, and bulk compression modulus respectively).

The specific heat capacity with constant stress $c_\sigma = (dQ/dT)_{\sigma_{ij}}$. Considering that $\dot{Q} = \dot{U} - (1/\rho_0)\sigma_{ij}\dot{\varepsilon}_{ij}$, $U = G + T\eta + (1/\rho_0)\sigma_{ij}\varepsilon^e_{ij}$, and assuming that the change in stresses equals zero, we have $\frac{dQ}{dt} = \frac{d}{dt}\left(G + T\eta - \frac{1}{\rho_0}\sigma_{ij}\varepsilon^p_{ij}\right)$. Then $c_\sigma = T\partial\eta_0/\partial T$, where it was considered that as follows from (1.16), (1.17), $\eta = \frac{\alpha_V}{\rho_0}\sigma + \eta_0$, $\eta_0 = \frac{1}{\rho_0}\frac{\partial G_0}{\partial T}$, $\dot{\eta}_0 = \frac{\partial \eta_0}{\partial T}\dot{T}$, and we designate $\sigma = \sigma_{kk}/3$.

We obtain a relationship $\dot{\eta} = \frac{\alpha_V}{\rho_0}\dot{\sigma} + \frac{c_\sigma}{T}\dot{T}$, which by substituting in (1.8) we arrive at a thermal conductivity equation of the form

$$\rho_0 c_\sigma \dot{T} + \alpha_V \dot{\sigma} T = t_{ij}\dot{\varepsilon}^p_{ij} + \Lambda\dot{\omega}^2 - \operatorname{div}\mathbf{q}. \tag{1.17}$$

We assume that material characteristics K and μ depend on damage parameter ω thus:

$$K = K_0(1 - \omega),\ \mu = \mu_0(1 - \omega) \tag{1.18}$$

(K_0 and μ_0 are bulk modulus and shear modulus of undamaged material which may depend on temperature, pressure, and other parameters [2]). Here it is assumed that ω varies from zero in entirely undamaged material to one in totally damaged material in which the supporting capacity equals zero. By using (1.15) and (1.16) we write

$$\sigma' = K_0\left(\varepsilon_{kk} - \alpha_V\ (T - T_0) - \frac{\Lambda}{3}\int_0^{\omega} \frac{\partial \dot{\omega}}{\partial \sigma} d\omega\right),\ S'_{ij} = 2\mu_0\left(e_{ij} - \varepsilon^p_{ij}\right) \tag{1.19}$$

($S'_{ij} = S_{ij}/(1 - \omega)$, $S_{ij} = \sigma_{ij} - \sigma\delta_{ij}$ are stress tensor deviator components, $e_{ij} = \varepsilon_{ij} - \frac{1}{3}\varepsilon_{kk}\delta_{ij}$ are strain tensor deviator components). In addition it is assumed that plastic flow is incompossible: $\varepsilon^p_{kk} = 0$.

For material with characteristics (1.18) there is the following property. If effective stress tensor $\sigma'_{ij} = \sigma_{ij}/(1 - \omega)$ is introduced into consideration, then in relation to it set

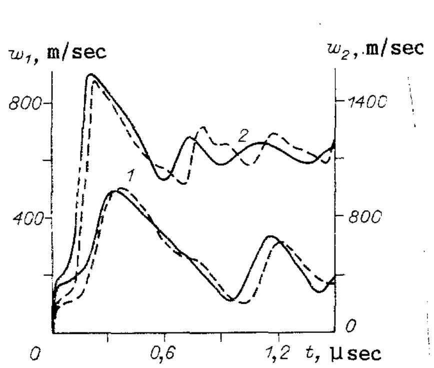

Fig. 1

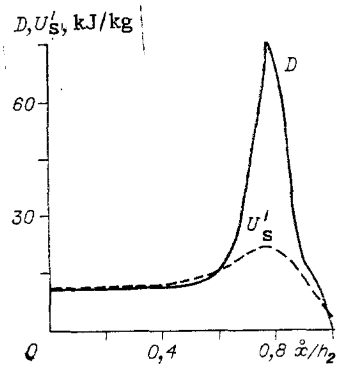

Fig. 2

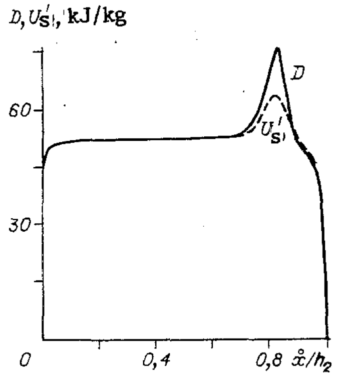

Fig. 3

of Eqs. (1.19) describes material behavior as it were without damage. This material may be considered as one whose mechanical properties do not change during deformation but are under the action of effective stresses σ'_{ij}. Therefore it is natural to apply the same approaches to this material as are used for describing classical elastoplastic materials. Thus we use a model for elastoplastic flow of the Prandtl–Reis type [1, 2]

$$(S'_{ij})^{\nabla} + \lambda S'_{ij} = 2\mu_0 \dot{e}_{ij},$$

where the symbol ∇ signifies Yaumanov derivative of the tensor component; λ is determined from the Mises plasticity condition $S'_{ij}S'_{ij} \leqslant \frac{2}{3}Y^2$; λ = 0 in an elastic region and $\lambda = \frac{3\mu_0 S'_{ij}\dot{e}_{ij}}{Y^2} H(S'_{ij}\dot{e}_{ij})$ in a region of plastic flow. Here Y is material yield strength; H(x) is Heavyside function. Taking account of strain anisotropy (which is provided by introducing active stress tensor t_{ij} [15]) the rule for flow and the Mises condition have the form

$$(\tau'_{ij})^{\nabla} + \lambda\tau'_{ij} = 2\mu_0\dot{e}_{ij}, \quad \tau'_{ij}\tau'_{ij} \leqslant \frac{2}{3}Y^2,$$

where $\tau_{ij} = S_{ij} - \rho_0 \partial G/\partial \varepsilon^p_{ij} = S_{ij} + \Gamma\varepsilon^p_{ij}$, $\tau'_{ij} = \tau_{ij}/(1-\omega)$.

The process of damage accumulation is written by a kinetic equation of the Tuler–Butcher type [9]

$$\dot{\omega} = B(\sigma' - \sigma_*)^m H(\sigma' - \sigma_*)$$

(B and m are material parameters). Presence of function H(x) is connected with the assumption that with values of tensile stress σ below a certain threshold value $\sigma_* \geq 0$ damage does not appear in the material and it does not develop.

We shall assume that the yield strength Y, shear modulus μ_0, and bulk compression K_0 depend as follows on temperature, pressure, and other parameters of state (Steinberg–Guinan model [2]):

$$\begin{gathered} Y = Y_0(1+\beta\varepsilon^p_u)^n\left(1 - b\sigma(\rho_0/\rho)^{1/3} - h(T-T_0)\right), \\ Y_0(1+\beta\varepsilon^p_u)^n \leqslant Y_{\max}, \quad Y_0 = 0 \text{ for } T > T_m, \\ T_m = T_{m0}(\rho_0/\rho)^{2/3}\exp(2\gamma_0(1-\rho_0/\rho)), \quad \mu_0 = \mu_{00}(1 - b\sigma(\rho_0/\rho)^{1/3} - \\ - h(T-T_0)) \end{gathered} \tag{1.20}$$

($\varepsilon^p_u = \sqrt{2\varepsilon^p_{ij}\varepsilon^p_{ij}/3}$ is plastic strain tensor intensity, $Y_0, Y_{\max}, \mu_{00}, T_{m0}, \beta, n, h, b, \gamma_0$ are material constants). We also assume that $\sigma_* = \sigma^0_* Y/Y_0$.

Thus, fundamental equations have the form

$$\sigma' = K_0\left(\varepsilon_{kk} - \alpha_V(T - T_0) - \frac{\Lambda}{3}\int_0^\omega \frac{\partial\dot\omega}{\partial\sigma}d\omega\right),$$

$$(\tau'_{ij})^\nabla + \lambda\tau'_{ij} = 2\mu_0\dot e_{ij},\ \tau'_{ij}\tau'_{ij} \leqslant \frac{2}{3}Y^2,\ \rho c_\sigma\dot T + \alpha_V\dot\sigma T = \tau_{ij}\dot\varepsilon^p_{ij} + \Lambda\dot\omega^2 - \operatorname{div}\mathbf{q},$$

$$\dot\omega = B(\sigma' - \sigma_*)^m H(\sigma' - \sigma_*),\ \tau_{ij} = S_{ij} + \Gamma\varepsilon^p_{ij},\ \tau'_{ij} = \frac{\tau_{ij}}{1-\omega},\ \sigma' = \frac{\sigma}{1-\omega}, \tag{1.21}$$

where material constants are in accordance with (1.20).

2. Failure Criterion. As a criterion of failure we take the condition of reaching specific dissipation which for the material model in question is

$$D = \int_0^t \frac{1}{\rho}\left(\tau_{ij}\dot\varepsilon^p_{ij} + \Lambda\dot\omega^2 - \frac{\mathbf{q}\operatorname{grad}T}{T}\right)dt, \tag{2.1}$$

of some limiting value D_*. This energy criterion for gailure makes it possible in principle to describe the process of failure both by the mechanism of microstructural damage accumulation which occurs for example with spalling failure in tensile waves (here the main contribution in (2.1) apart from the magnitude of mechanical dissipation $\tau_{ij}\dot\varepsilon^p_{ij}/\rho$, is made by the term $\Lambda\dot\omega^2/\rho$, i.e., the magnitude of continuous failure), and by shear arising for example in problems of piercing an obstacle of finite thickness by a striker with a flat leading section. As is well known, in the least case a narrow zone of intense adiabatic shear develops in the obstacle in areas of stress concentration. The work of plastic deformation is almost entirely converted into heat which due to the high local strain rates cannot propagate over a marked distance from the zone of developed plastic strain. As a result of this the temperature in this zone increases, considerable temperature gradients develop, which causes additional plastic flow and concentration of local plastic strains, and it leads finally to punching of a "plug" from the obstacle. With failure by shear a specific contribution in (2.1) is given by the terms $\tau_{ij}\dot\varepsilon^p_{ij}/\rho$ and $-\mathbf{q}$ grad $T/\rho T$. The latter, which is the magnitude of thermal dissipation, for the Fourier thermal conductivity rule $\mathbf{q} = -\kappa$ grad T has the form $\kappa(\text{grad } T)^2/\rho T$.

It is noted that different empirical energy criteria are often used in order to describe failure both with creep [16], failure by shear [17], with spalling [18, 19], and as a single failure criterion for the mechanism of shear and separation [20], and it give satisfactory results.

In [16] a condition of failure with creep is reaching dissipation energy

$$A = \int_0^t \sigma_{ij}\eta_{ij}dt \tag{2.2}$$

of some limiting value A_* (η_{ij} are creep strain rate tensor components), in [17] with numerical modelling of the problem for piercing of a "plug" from an obstacle use is made of a criterion of limiting specific work of plastic deformation

$$A_p = \int_0^t \frac{1}{\rho} S_{ij}\dot\varepsilon^p_{ij}dt \leqslant A^*_p, \tag{2.3}$$

and in [18-20] use is made of a Huber—Mises—Genk criterion of limiting specific shape change energy

$$U_s = \int_0^t \frac{1}{\rho} S_{ij}\dot\varepsilon_{ij}dt \leqslant U^*_s. \tag{2.4}$$

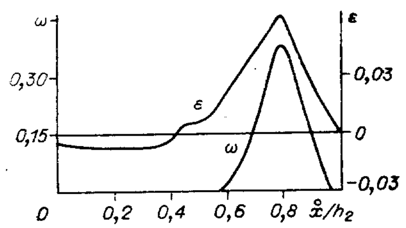

Fig. 4

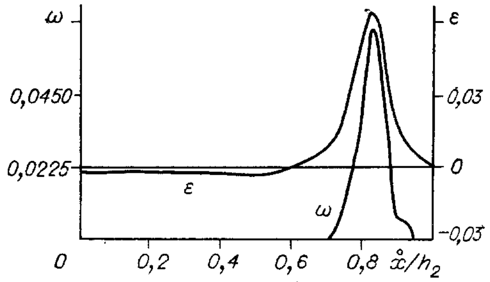

Fig. 5

It can be seen that in the case of simplifying model (1.21) (not introducing internal parameters of state ω and ε^p_{ij} and ignoring thermal effects) criterion (2.1) conforms with (2.3) and it is little different from (2.2) and (2.4). However, failure criteria for specific dissipation (2.1) have a clear thermodynamic basis which it is not possible to say about empirical criteria (2.2)-(2.4).

3. Evaluation of the model was accomplished in solving the problem of plane impact of two plates for experimental conditions in [21].

Since the thickness of the plates is small compared with their dimensions and the characteristic time of the process is ~2-5 μsec [21] (the time for some travel of an elastic wave through the thickness of the target plate), the problem of impact was solved in a unidimensional mathematical arrangement (uniaxial deformed state) and an adiabatic approximation (div **q** = 0). Here the equations of continuity, pulse, and internal energy are written in a Cartesian coordinate system Oxyz (axis x is perpendicular to the surfaces of the plates) as follows:

$$\dot{\rho}/\rho = -\dot{\varepsilon},\ \dot{v} = (1/\rho)\partial(S+\sigma)/\partial x,\ \rho c_\sigma \dot{T} + \alpha_V \sigma \dot{T} = (3/2)S\dot{\varepsilon}^p + \Lambda\dot{\omega}^2.$$

Here v = v_x is velocity: $\dot{\varepsilon} = \dot{\varepsilon}_{xx} = \partial v/\partial x$; $\dot{\varepsilon}^p = \dot{\varepsilon}^p_{xx}$; S = S_{xx} is stress deviator tensor component; the rest of the variables conform with those introduced previously. In addition, strain anisotropy of the material is ignored (Γ = 0) and it is considered that $S_{yy} = S_{zz} = -S_{xx}/2$, $\dot{\varepsilon}^p_{yy} = \dot{\varepsilon}^p_{zz} = -\dot{\varepsilon}^p_{xx}/2$, since S_{kk} = 0 and $\dot{\varepsilon}^p_{kk}$ = 0.

$$\dot{\sigma}' = K_0\left(\dot{\varepsilon} - \alpha_V\dot{T} - \frac{\Lambda}{3}\dot{\omega}\frac{\partial\dot{\omega}}{\partial\sigma}\right),\ \dot{S}' + \lambda S = \frac{4}{3}\mu_0\dot{\varepsilon},\ |S'| \leqslant \frac{2}{3}Y,$$

$$\dot{\omega} = B(\sigma' - \sigma_*)^m H(\sigma' - \sigma_*).$$

Material characteristics Y, μ_0, K_0 are found from (1.20). Failure is analyzed on the basis of limiting specific dissipation (2.1).

There are the following initial conditions: $v = V_0$, $\rho = \rho_{01}$, $\sigma = S = 0$, $T = T_0$ $(-h_1 \leqslant \mathring{x} \leqslant 0)$ for the striker and $v = 0$, $\rho = \rho_{02}$, $\sigma = S = 0$, $T = T_0$ $(0 \leqslant \mathring{x} \leqslant h_2)$ for the target. Here $\mathring{x} = x|_{t=0}$ is initial Lagrange coordinate; h_1, h_2 are striker and target thickness. Boundary conditions at the free surfaces of the plates $(\mathring{x} = -h_1,\ \mathring{x} = h_2)$: $\sigma + S = 0$. Boundary conditions at the contact surface $\mathring{x} = 0$: $v^+ = v^-$, $(\sigma + S)^+ = (\sigma + S)$ for compressive forces $(\sigma + S)^+ = (\sigma + S)^- < 0$ and conditions for the free surfaces σ + S = 0 in the opposite case.

The same as at the contact surface $\mathring{x}$ = 0 boundary conditions are set at the surfaces of spalling failure introduced during the calculation in sections of the target where failure criterion (2.1) is fulfilled.

The problem is solved in a Lagrange calculation grid by an explicit finite difference scheme [1]. An algorithm for constructing surfaces of spalling failure in a plate based on the procedure of rebuilding the Lagrange grid in the failure surface and the procedure for converting the parameters of state to a new grid are given in [18], and the method for numerical realization of boundary conditions at the contact surface of the plates and the failure surface is given in [22].

The striker material is aluminum, and the obstacle is titanium [21]. Material characteristics are taken from [2]: for aluminum ρ_0 = 2780 kg/m^3, K_0 = 79.06 GPa, μ_0 = 27.6 GPa, Y_0 = 0.29 GPa. Y_{max} = 0.68 GPa, T_{m0} = 1220 K, β = 125, b = 0.065 GPa^{-1}, n = 0.1, h = $6.2 \cdot 10^{-4}$ K^{-1}, α_V = $6.72 \cdot 10^{-5}$ K^{-1}, γ_0 = 1.97. c_σ = 924.3 J/(kg·K), κ = 230 W/(m·K); for titanium ρ_0 = 4530 kg/m^3, K_0 = 123.4 GPa, μ_0 = 43.4 GPa, Y_0 = 0.71 GPa, Y_{max} = 1.45 GPa, T_{m0} = 2260 K, β = 780, b = 0.0115 GPa^{-1}, w = 0.065, h = $6.2 \cdot 10^{-4}$ K^{-1}, α_V = $2.52 \cdot 10^{-5}$ K^{-1}, γ_0 = 1.23, c_σ = 520.7 J/(kg·K), κ = 20 W/(m·K). Striker thickness h_1 = 2 mm, target thickness h_2 = 10 mm. Parameters of the model B, Λ, σ_*, and m are selected from comparison of the results in [21] with calculations: for aluminum B = $1.034 \cdot 10^{-3}$ (Pa·sec)$^{-1}$. Λ = 193.3 Pa·sec, m = 1, σ_*^0 = 0.145 GPa: for titanium B = $4.225 \cdot 10^{-4}$ (Pa·sec)$^{-1}$, Λ = 591.7 Pa·sec, m = 1, σ_*^0 = 1.065 GPa.

Shown in Fig. 1 is the dependence of the velocity of the rear surface of the target $w = v|^{\circ}_{x=h_2}$ on time for striker velocities V_0 = 660 (curve 1) and 1900 m/sec (curve 2, solid lines are the experiment [21], broken lines are the calculation). In calculations for titanium use was made of the value of limiting specific dissipation D_* = 75 kJ/kg. As can be seen from Fig. 1, the calculated results are in good agreement with experiments both for the time of forming spalled plates, their thickness, and with respect to spalling velocities. Divergence in the calculated and experimental values of the amplitude of elastic precursors is apparently connected with the fact that in the Shteinberg–Guinan model (1.20) no consideration is given in an explicit way to the dependece of material yield strength on strain rate which affects the very initial stage of the analysis in forming a compression wave propagating through the target.

Presented in Figs. 2 and 3 are the distribution of values of specific dissipation and relative specific shape change energy

$$U'_S = \int_0^t \frac{1}{\rho} S'_{ij} \dot{\varepsilon}_{ij} dt$$

at the instants of fulfilling failure criterion (2.1) for impact velocities V_0 = 660 and 1900 m/sec, respectively. It can be seen that the maximum U'_S is reached in those sections where the maximum specific dissipation D is achieved, although as a failure criterion the value of U'_S cannot be selected since the maximum U'_S increases markedly with an increase in V_0.

Shown in Figs. 4 and 5 are the distribution of damage parameter ω and strain ε at the instant of fulfilling criterion (2.1) for V_0 = 660 and 1900 m/sec. Maxima for ω and ε are reached in the same section as for the maximum D. However, with an increase in V^0 the limiting value of ω decreases markedly, and this means that it cannot serve as a criterion for spalling failure. Maximum tensile strains ε in sections of failure for V_0 = 660 and 1900 m/sec almost coincide.

Thus, a coherent model is built up for a damaged thermoelastoplastic body with internal parameters of state. Criteria are suggested for failure by limiting specific dissipation which make it possible in principle to describe failure under conditions of a complex stressed state both by a shear mechanism and by a separation mechanism as a result of accumulation of damage parameters in tensile areas which affect the stressed state. On the example of solving the problem for plane impact of plates it is shown that the model makes it possible to predict correctly the main features of the process and failure criteria suitable for describing spalling failure.

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